Stochastic Decision Petri Nets

by Florian Wittbold[1], Rebecca Bernemann[1], Reiko Heckel[2], Tobias Heindel[3], and Barbara König[1]

Florian Wittbold

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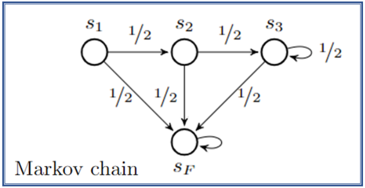
June 29, 2023

Interactivity in Stochastic Models



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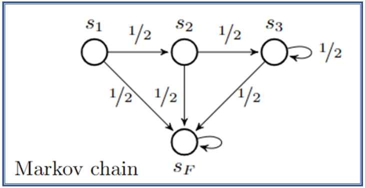

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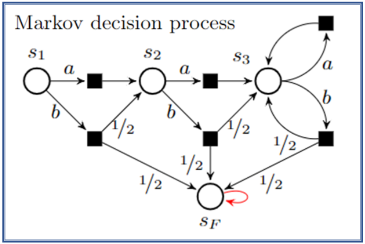

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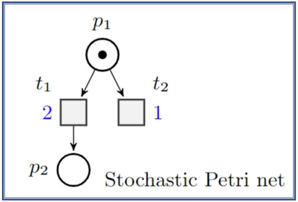

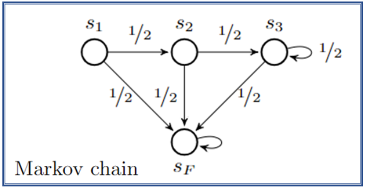

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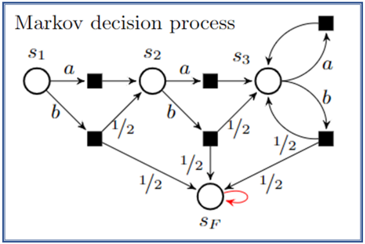

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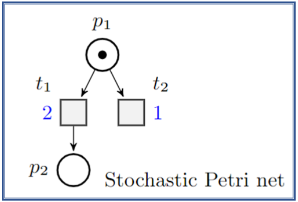

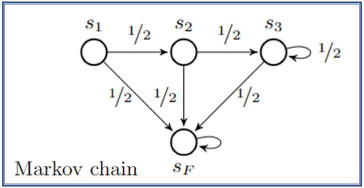

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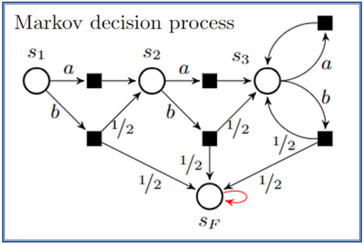


M. Beccuti, G. Franceschinis, and S. Haddad. *Markov decision Petri nets and Markov decision well-formed net formalisms.* In Petri Nets and Other Models of Concurrency. Springer, 2007.

Stochastic Decision Petri Nets

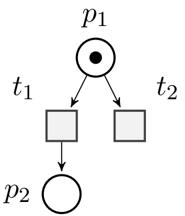
Definition

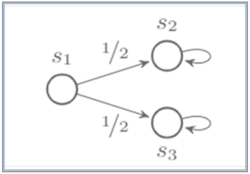
A **stochastic decision Petri net** (SDPN) is a tuple (N, C, R) where

- N is a stochastic Petri net with places P and transitions T,
- $C \subseteq T$ is a set of controllable transitions, and
- $R : \mathcal{P}(P) \to \mathbb{R}$ is a reward function.

Introducing Decisions

$C := \emptyset$





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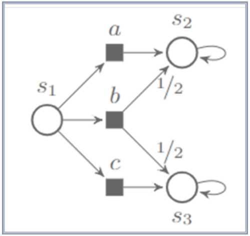
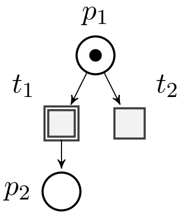
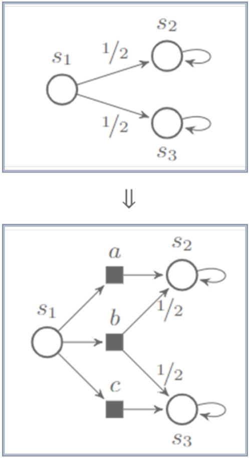


Introducing Decisions

$$C := \{t_1\}$$





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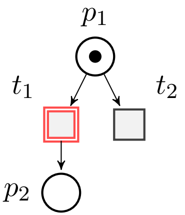
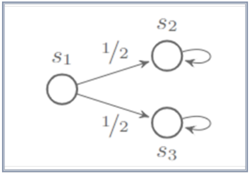


$$D = \{t_1\} \subseteq C$$



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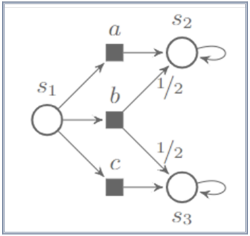
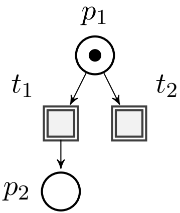
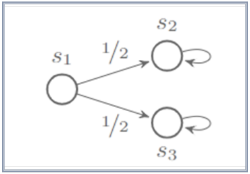


Introducing Decisions

$$C \coloneqq \{t_1, t_2\}$$





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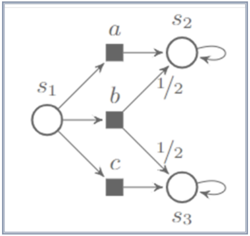
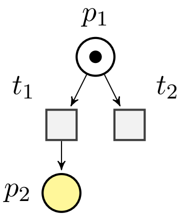
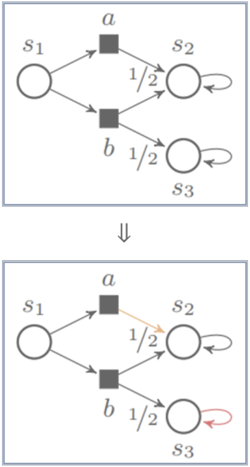


Introducing a Goal

$$R = [\{p_2\} : 1]$$





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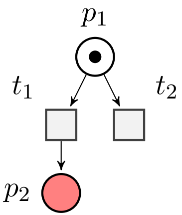
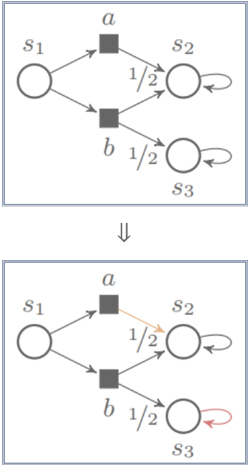
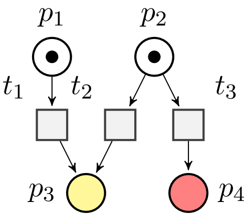
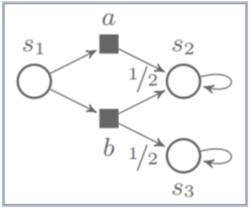




Introducing a Goal

$$R = [\{p_3\} : 1, \{p_3, p_4\} : -1]$$





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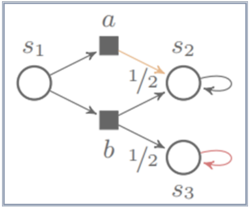

Value and Policy Problem

Definition (Value Problem $\mathcal{N}$-VAL)

Given a rational $p > 0$, a net $N \in \mathcal{N}$ and a policy $\pi \in \Psi$ for N, decide whether $\mathbb{V}^\pi > p$.

Definition (Policy Problem $\mathcal{N}$-POL)

Given a rational $p > 0$ and a net $N \in \mathcal{N}$, decide whether there exists a policy $\pi \in \Psi$ such that $\mathbb{V}^\pi > p$.

Results

Class of SDPNs	Value prob.	Policy prob.
Free-choice occ. nets	P	NP-cpl.

Table: Complexity of value and policy problems for specific classes of SDPNs under *constant* policies.

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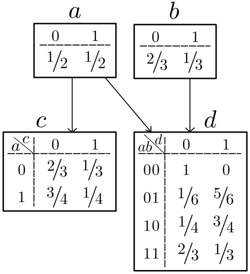
D-MAP Problem for Bayesian Networks

D-MAP: Given a Bayesian network with variables X, a rational number $p > 0$, disjoint subsets $E, F \subseteq X$, and evidence $e \in V_E$, does there exist $f \in V_F$ such that $\mathbb{P}(F = f, E = e) > p$.



$E \coloneqq \{c\}, F \coloneqq \{a\}, e \coloneqq 1$

J.D. Park and A. Darwiche. *Complexity results and approximation strategies for MAP explanations.* Journal of Artificial Intelligence Research, 21:101–133,2004.

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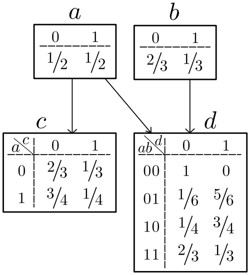

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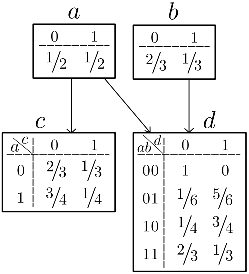
Interpreting Bayesian Networks as SDPNs

$$\text{D-MAP}^* \leq_p \text{SAFC-POL}$$



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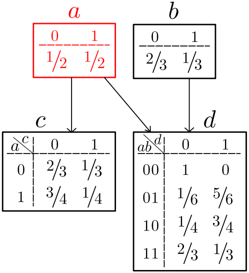

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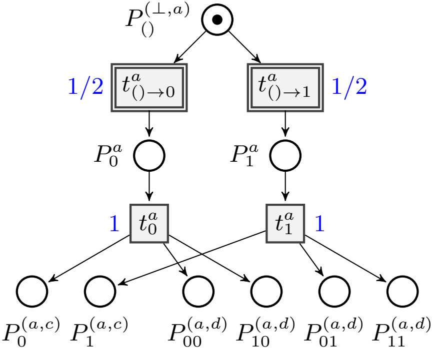

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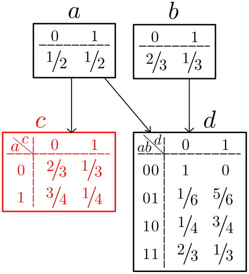

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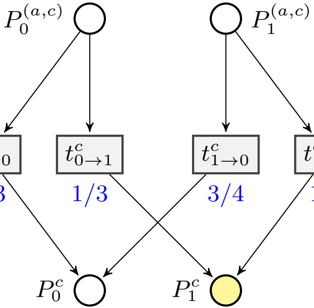
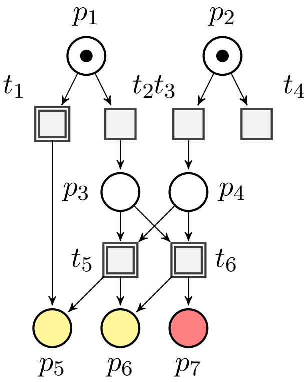


R. Bruni, H.C. Melgratti, and U. Montanari. *Bayesian network semantics for Petri nets.* Theoretical Computer Science, 807:95–113, 2020

A Partial-Order Algorithm for Safe and Acyclic Free-Choice SDPNs



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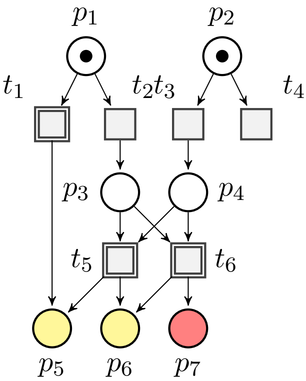

$\{t_1\} : 1$

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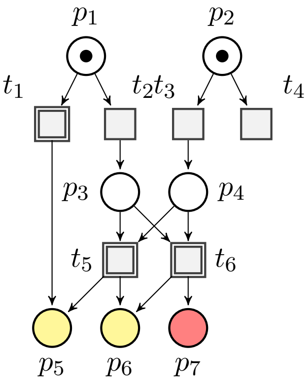

$$[\{t_1\} : 1, \{t_2, t_3, t_5\} : 1]$$

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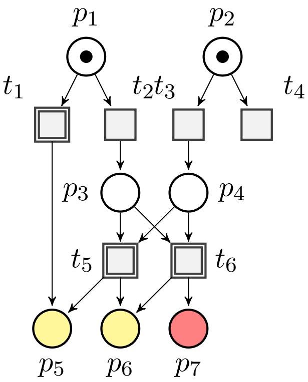

$$[\{t_1\} : 1, \{t_2, t_3, t_5\} : 1]$$

$$\mathbb{V}^D = \frac{\chi_{T\setminus D}(t_1)}{\chi_{T\setminus D}(t_1) + 1} + \frac{1}{2} \frac{1}{\chi_{T\setminus D}(t_1) + 1} \frac{\chi_{T\setminus D}(t_5)}{\chi_{T\setminus D}(t_5) + \chi_{T\setminus D}(t_6)}$$

S. Abbes and A. Benveniste. *True-concurrency probabilistic models:branching cells and distributed probabilities for event structures.* Information and Computation, 204(2):231–274, 2006

Recap

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Future Work

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